## BE SURE TO READ THE DIRECTIONS PAGE \& MAKE YOUR NOTECARDS FIRST!! <br> Part I: Unlimited and Continuous! (21 points)

For \#1-4 below, find the limits, if they exist.(\#1-13 are 1 pt each)

1) $\lim _{x \rightarrow 4} \frac{2 x^{3}-7 x^{2}-4 x}{x-4}$
2) $\lim _{x \rightarrow 9} \frac{\sqrt{x}-3}{9-x}$
3) $\lim _{x \rightarrow 1} \frac{x^{2}-2 x-5}{x+1}$
4) $\lim _{x \rightarrow-2} \frac{x^{3}+8}{x+2}$

For \#5-7, explain why each function is discontinuous and determine if the discontinuity is removable or nonremovable. (Use limit notation!)
5) $g(x)= \begin{cases}2 x-3, & x<3 \\ -x+5, & x \geq 3\end{cases}$
6) $b(x)=\frac{x(3 x+1)}{3 x^{2}-5 x-2}$
7) $h(x)=\frac{\sqrt{x^{2}-10 x+25}}{x-5}$

For \#8-13, determine if the following limits exist, based on the graph below of $p(x)$. If the limits exist, state their value. Note that $x=-3$ and $x=1$ are vertical asymptotes.

8) $\quad \lim _{x \rightarrow 1^{-}} p(x)$
9) $\quad \lim _{x \rightarrow-3^{-}} p(x)$
10) $\lim _{x \rightarrow 2} p(x)$
11) $\lim _{x \rightarrow 3^{-}} p(x)$
12) $\lim _{x \rightarrow 3^{+}} p(x)$
13) $\lim _{x \rightarrow-1} p(x)$

3 14) Consider the function $f(x)=\left\{\begin{array}{ll}x^{2}+k x & x \leq 5 \\ 5 \sin \left(\frac{\pi}{2} x\right) & x>5\end{array}\right.$,
Find the value of $k$ in order for the function to be continuous at $\mathrm{x}=5$.

2 15) Consider the function $f(x)=\left\{\begin{array}{ll}\frac{\sin x}{x} & x \neq 0 \\ k & x=0\end{array}\right.$.
Find the value of $k$ in order for the function to be continuous at $\mathrm{x}=0$.

Use the graph of f(x), shown below, to answer \#16-18. (1 pt each).

16) For what value of a is $\lim _{x \rightarrow a} f(x)$ nonexistent?
17)
$\lim _{x} f(x)=$ $x \rightarrow \infty$
18)

$$
\lim _{x \rightarrow-\infty} f(x)=
$$

## Part II: Designated Deriving! (38 points)

$\square$

1) $\lim _{h \rightarrow 0} \frac{\tan ^{-1}(1+h)-\frac{\pi}{4}}{h}=$

1 2) $\lim _{h \rightarrow 0} \frac{\sec (\pi+h)-\sec (\pi)}{h}=$
$h \rightarrow 0$

For \#3-8, find the derivative.
2
3) $y=\ln \left(1+e^{x}\right)$
2 4) $y=\csc (1+\sqrt{x})$

4
5) $y=\left(\tan ^{2} x\right)\left(3 \pi x-e^{2 x}\right)$

2 6) $y=\sqrt[7]{x^{3}-4 x^{2}}$

3
7) $f(x)=(x+1) e^{3 x}$
3
8) $f(x)=\frac{e^{x / 2}}{\sqrt{x}}$

2 9) Consider the function $f(x)=\sqrt{x-2}$. On what intervals are the hypotheses of the Mean Value Theorem satisfied?
10) If $x y^{2}-y^{3}=x^{2}-5$, then $\frac{d y}{d x}=$

2 11) The distance of a particle from its initial position is given by $s(t)=t-5+\frac{9}{(t+1)}$, where $s$ is feet and $t$ is minutes. Find the velocity at $t=1$ minute in appropriate units.

Use the table below for \#12-13.

| X | $f(x)$ | $g(x)$ | $f^{\prime}(x)$ | $g\left(^{\prime} x\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 | 2 | 5 | $1 / 2$ |
| 3 | 7 | -4 | $\frac{3}{2}$ | -1 |

2. 12) The value of $\frac{d}{d x}(f \cdot g)$ at $\mathrm{x}=3$ is 2 13) The value of $\frac{d}{d x}\left(\frac{f}{g}\right)$ at $\mathrm{x}=1$ is

In \#14-15, use the table below to find the value of the first derivative of the given functions for the given

| X | $f(x)$ | $g(x)$ | $f^{\prime}(x)$ | $g\left(^{\prime} x\right)$ |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 3 | 2 | 0 | $3 / 4$ |
| 2 | 7 | -4 | $\frac{1}{3}$ | -1 |

14) $[f(x)]^{2}$ at $\mathrm{x}=2$ is

2 15) $f(g(x))$ at $\mathrm{x}=1$ is
16) Let f be the function defined by $f(x)=\frac{x+\sin x}{\cos x}$ for $-\frac{\pi}{2}<x<\frac{\pi}{2}$.

2 (a) State whether $f$ is an even function or an odd function. Justify your answer.
(b) Find $f^{\prime}(x)$.
(c) Write an equation for the line tangent to the graph of $f$ at the point $(0, f(0))$.

## Part III: Derived and Applied! (27 points)

For \#1-3, find all critical values, intervals of increasing and decreasing, any local extrema, points of inflection, and all intervals where the graph is concave up and concave down.

4 1) $f(x)=\frac{5-4 x+4 x^{2}-x^{3}}{x-2}$
3 2) $y=3 x^{3}-2 x^{2}+6 x-2$
3 3) $f^{\prime}(x)=5 x^{3}-15 x+7$
4) The graph of the function $y=x^{5}-x^{2}+\sin x$ changes concavity at $\mathrm{x}=$

3 5) Find the equation of the line tangent to the function $y=\sqrt[4]{x^{7}}$ at $x=16$.
6) For what value of x is the slope of the tangent line to $y=x^{7}+\frac{3}{x}$ undefined?
7)


The balloon shown is in the shape of a cylinder with hemispherical ends of the same radius as that of the cylinder. The balloon is being inflated at the rate of $261 \pi$ cubic centimeters per minute. At the instant the radius of the cylinder is 3 centimeters, the volume of the balloon is $144 \pi$ cubic centimeters and the radius of the cylinder is increasing at the rate of 2 centimeters
per minute. (The volume of a cylinder with radius $r$ and height $h$ is $\pi r^{2} h$, and the volume of a sphere with radius $r$ is $\frac{4}{3} \pi r^{3}$.)

3 (a) At this instant, what is the height of the cylinder?
3 (b) At this instant, how fast is the height of the cylinder increasing?
8)


A ladder 15 feet long is leaning against a building so that end X is on level ground and end Y is on the wall as shown in the figure. X is moved away from the building at a constant rate of $1 / 2$ foot per second.

3 (a) Find the rate in feet per second at which the length OY is changing when X is 9 feet from the building.
3 (b) Find the rate of change in square feet per second of the area of triangle XOY when X is 9 feet from the building.

## Part V: Integral to Your Success! (31 points)

3 1) $\int_{-8}^{-1} \frac{x-x^{2}}{2 \sqrt[3]{x}} d x$
3 2) $\int_{-\pi / 6}^{\pi / 6} \sec ^{2} x d x$

1 3) $\frac{d}{d x} \int_{1}^{x} \sqrt[4]{t} d t$
2 4) $\frac{d}{d x} \int_{\sin (4 x)}^{0} e^{t} d t$

5) $\int \frac{x^{3}}{\sqrt{1+x^{4}}} d x$
2
6) $\int \frac{\csc ^{2} x}{\cot ^{3} x} d x$

2 7) $\int \sqrt{\tan x} \sec ^{2} x d x$

2 8) What are all the values of $k$ for which $\int_{2}^{k} x^{5} d x=0$ ?
9) What is the average value of $y=x^{3} \sqrt{x^{4}+9}$ on the interval [0, 2]?
10) If $\int_{a}^{b} g(x) d x=4 a+b$, then $\int_{a}^{b}[g(x)+7] d x=$
11) The function $f$ is continuous on the closed interval [1,9] and has the values given in the table. Using the subintervals $[1,3],[3,6]$, and $[6,9]$, what is the value of the trapezoidal
approximation of $\int_{1}^{9} f(x) d x$ ?

| x | 1 | 3 | 6 | 9 |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{f}(\mathrm{x})$ | 15 | 25 | 40 | 30 |

12) The table below provides data points for the continuous function $y=h(x)$.

| x | o | 2 | 4 | 6 | 8 | 10 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{~h}(\mathrm{x})$ | 9 | 25 | 30 | 16 | 25 | 32 |

Use a right Riemann sum with 5 subdivisions to approximate the area under the curve of $y=h(x)$ on the interval [0, 10].
13) A particle moves along the x -axis so that, at any time $t \geq 0$, its acceleration is given by $a(t)=6 t+6$. At time $t=0$, the velocity of the particle is -9 , and its position is -27 .
(a) Find $v(t)$, the velocity of the particle at any time $t \geq 0$.
(b) For what values of $t \geq 0$ is the particle moving to the right?
(c) Find $x(t)$, the position of the particle at any time $t \geq 0$.

## Part VI: Apply Those Integrals! (35 points)

For \#1-2, find the general solution to the given differential equation.
3 1) $\frac{d y}{d x}=\frac{3 y}{2+x}$
3 2) $\frac{d y}{d x}=y \sin x$

4 3) Find the particular solution to the differential equation $\frac{d u}{d v}=u v \sin v^{2}$ if $u(0)=1$.
4) The shaded regions, $\mathrm{R}_{1}$ and $\mathrm{R}_{2}$ shown above are enclosed by the graphs of $f(x)=x^{2}$ and $g(x)=2^{x}$.

1 (a) Find the x - and y-coordinates of the three points of intersection of the graphs of $f$ and $g$.
(b) Without using absolute value, set up an expression involving one or more integrals that gives the total area enclosed by the gra
$f$ and $g$. Do not evaluate.
(c) Without using absolute value, set up an expression involving one
 or more integrals that gives the volume of the solid generated by revolving the region $\mathrm{R}_{1}$ about the line $\mathrm{y}=5$. Do not evaluate.
5) Let R be the region in the first quadrant under the graph of $y=\frac{1}{\sqrt{x}}$ for $4 \leq x \leq 9$.

(a) Find the area of R.
(b) If the line $\mathrm{x}=\mathrm{k}$ divides the region R into two regions of equal area, what is the value of k ?
(c) Find the volume of the solid whose base is the region R and whose cross sections cut by planes perpendicular to the x -axis are squares.
6) Below is the graph of $f$ which consists of 3 line segments and a semicircle. Let $g$ be the function given by

$$
g(x)=\int_{-3}^{x} f(t) d t
$$

(a) Find $g(0)$ and $g^{\prime}(0)$
(b) Find all values of $x$ in the open interval $(-5,4)$ at which $g$ attains a relative minimum. Justify your answer.
(c) Find the absolute minimum value of $g$ on the closed interval $[-5,4]$. Justify your answer.
(d) Find all values of $x$ in the open interval $(-5,4)$ at which the graph of $g$ has a point of inflection.

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